Electromagnetic Modeling of Multi-Layer Microwave Circuits by the Longitudinal Decomposition Approach

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Abstract — **A system approach based on the transverse-resonance technique and well-equipped modeling system is used to calculate the eigen-mode spectrum of multi-conductor lines with piece-wise continuous coordinate boundaries of their cross-section. The exact models of the multi-layer circuits are built with the mode-matching and generalized S-matrix techniques. Viability of the created program tools is demonstrated with the examples of a multi-turn loop inductor and a third-order low-pass filter.**

I. Introduction

Modeling of multi-layer circuits is now a very important problem in applied electromagnetics [1, 2]. Instead of the layer-to-layer circuit analysis [2], it is possible to consider the circuit structure in a longitudinal direction across the layers. In doing so, the circuit structure may be interpreted as a sequence of junctions of some lines of complicated cross-sections. One of the most popular techniques for computing the eigen-modes of such transmission lines is the transverse resonance method. It is based on the decomposition of a line cross-section into fragments with the known scattering properties. The availability of modeling systems equipped with tools for the electromagnetic assembling of scatterers by the generalized *S*-matrix technique as well as an advanced library of waveguide key elements makes it possible to solve efficiently fairly large classes of eigen-mode problems.

The goal of the given work is to demonstrate a unified approach to the calculation of the full-wave spectrums of complicated lines and to analyze the corresponding scattering problems. The modified version of the modeling system [3] is employed both at the stage of building the mode bases for various lines and at the final stage of calculating the frequency response of an integrated circuit. The proposed approach allows taking into account the finite thickness of conductors that was a problem in [2] and at using the commercial software products built on the method of moments.

The main philosophy behind the proposed approach is demonstrated by the analysis of a loop-type element shown in Fig. 1. According to the proposed approach, the loop topology is represented as a set of regular sections of multi-conductor transmission lines with a rectangular enclosure filled by an isotropic dielectric medium. The cross-sections of the obtained lines are shown in Fig. 2. Cascaded junctions of such lines forms the analyzed structure. Note that a segment of line *B* in Fig. 2 forms in particular two via holes.

The calculation of the characteristics of the loop requires a preliminary analysis of the full spectrum of TEM, TE and TM modes in separate lines to use further the mode-matching method for the calculation of the *S*-matrixes of the plane junctions of those lines.

II. TE and TM Mode Basis

Algorithms for obtaining the desired spectrum of TE and TM modes are based on the solutions of two-dimensional homogeneous boundary-value problems formulated in terms of H_z or E_z field components of TE or TM modes, correspondingly. The desired solutions are obtained in two steps. Considering the separate line cross-section as a set of key-elements (such as shortings, *N*-furcations, steps, and so on) in a *y*-directed parallel-plate waveguide, we have to calculate their generalized *S*-matrices. Following that, the assembling of the key-elements in a certain order is performed by the generalized *S*-matrix technique. The homogeneous matrix equation or a system of such equations is the result of this step. The unknowns in those equations are the amplitudes of the forward and backward modes traveling in the *y*-direction in parallel-plate waveguide sections (partial domains) between the key-elements. Nontrivial solutions of the homogeneous problems describe the desired spectrum of TE or TM modes.

The above homogeneous matrix equations may be formulated relative to the amplitudes of modes traveling in one of chosen partial domains (for example, in the first or last domain for the lines in Fig. 2), or in several domains, or in all the domains. The *first* algorithm generates a small-order determinant but it is characterized by a steep frequency response in the vicinity of the determinant zeros. The other disadvantage is connected with a possible field localization of one of the modes (especially, TM modes) in a domain other than a chosen one. At a weak electromagnetic coupling of the chosen domain with the domain of the field localization, some of the determinant zeros may be omitted.

The *second* algorithm based on forming a homogenous matrix equation connected with the mode amplitude vectors in some of the partial domains are more time-consuming but the frequency dependence of the corresponding determinant becomes smoother. In both the above-described algorithms, the problem of parasitic poles near the cutoff frequencies of some domains may appear. It makes the process of searching for the line eigen-modes more difficult.

The *third* algorithm is built with taking into account the couplings between the vectors of modes traveling in all the partial domains. Since in the general case not all the partial domains are coupled with each other, the matrix operator contains a great number of zero elements. On the one hand, it leads to a more time-consuming algorithm. On the other hand, the determinants generated by such an algorithm do not have parasitic poles, their frequency dependence is fairly smooth, and determinant zeros are localized easily.

All the above-described algorithms are three-steps ones. In the first step, a rough localization of zeros of the corresponding determinant is performed. It is based on searching the frequencies at passing of which the complex-valued determinant changes its sign. In the second step, the rough localized zeros are refined by Newton's method. Basing on the results of the latter, the third step consists in the solution of the homogenous matrix equations to obtain the amplitude vectors of modes traveling in each partial domain of the considered line cross-section.

III. TEM Mode Basis

The transverse resonance method can be also used for the determination of the TEM modes fields with taking into account their possible degeneration in the case of multi-conductor lines. In contrast to TE and TM modes, the problem on TEM modes may be formulated as a three-dimensional one. For that, a half-wave resonator built by a multi-conductor line has to be considered. In doing so, the TEM mode fields are obtained by superposition of the TE and TM modes traveling in the *y*-direction of such a resonator. Such an approach was repeatedly used in the case of single-conductor transmission lines with inner conductors of various forms (see, for instance, [4]).

Following to the above concept, we come to the problem of determination of the cross (to the resonator axis) fields of the half-wave resonator's TEM oscillations at the known resonant frequency $f_0 = c/2d$ where c is the free-space light velocity and d is the resonator length. Unlike the problem on the higher-order mode spectrum, the transverse resonance condition is formulated for the superposition of TE_{m1} and TM_{m1} modes traveling in the *y*-directed partial rectangular waveguides. If the line being analyzed has N inner conductors, then the problem of determination of the eigen-fields of N-times degenerated TEM modes appears.

To eliminate the degeneration of the eigen-frequencies of half-wave TEM mode resonators, an artificial approach was used. It consists in an insertion of discontinuities at resonator ends. As discontinuities, steps of a small height $\Delta^{(i)}/2$ were inserted symmetrically to both resonator ends but only for some partial domains forming the line cross-section. TEM modes with a various field structure will react to such steps in different ways. Due to that, a splitting of the resonator's degenerated eigen-frequencies is expected.

Let us denote the spectrum of split eigen-frequencies as $f^{(k)}, k = 1,...N$. It is easy to notice that $f_0 < f^{(k)} < f^{(s)}_{\max} = c/2(d - \Delta^{(s)}_{\max})$ where $\Delta^{(s)}_{\max}$ corresponds to the s-th domain with a maximum step height value. Finally, the problem is reduced to the search of N zeros of the corresponding matrix determinant over the range $f_0 < f < f^{(s)}_{\max}$. As experience showed, the choice of equal-step resonator loads with $\Delta^{(s)} = \Delta$ makes the problem on determining the spectrum of TEM modes much easier.

For the real choice of Δ, one should take into account two factors. On the one hand, it is desirable to choose the value of Δ as small as possible, in order to minimize the difference of the object being considered from the non-perturbed $\lambda_0/2$-resonator in general, and in order to reduce the level of superposed TE_{mn} and TM_{mn} modes with $n = 3,5,...,$ in particular. The latter modes are generated due to the steps inserted to a part of partial domains. Let us note that already at $\Delta/d \approx 10^{-2}$ the amplitudes of TE_{m3} and TM_{m3} modes turn out to be three or four orders lower than the amplitudes of TE_{m1} and TM_{m1} modes and those modes may be ignored.

At each split frequency the resonator field corresponds to a separate TEM mode having no E_z and H_z field components. Really, they are about $10^{-4} \div 10^{-5}$ in comparison with the amplitudes of transverse components. Based on this fact, the relationships between the amplitudes of TE_{m1} and TM_{m1} modes providing the conditions E_z=0 and H_z=0 are easily set. Due to them, the TEM mode field may be presented as the superposition of "equivalent" TE_{m1} modes only, the amplitudes of which are combinations of the ones for the initial set of TE_{m1} and TM_{m1} modes.

Let's point out that the technique of searching for the degenerated TEM modes used here simultaneously deals away with the well-known orthogonalization problem, since the found field distributions are orthogonal accurate to within $10^{-5} \div 10^{-6}$. To check the adequacy of the data being obtained a set of internal criteria was developed. For example, integrating the TEM field components E_x or E_y along a way from one a line enclosure wall to other allows estimating a level of "potentiality" of the found TEM mode fields (such integrals should be zero). It turned out that corresponding conditions are met accurate to $10^{-6} \div 10^{-7}$ if 10 - 15 "equivalent" TE_{m1} modes are taken into account within partial domains.

The viability of the created algorithm for calculating the TEM modes fields are demonstrated by Fig. 3 where the brightness pictures of E_x-component distributions are presented only for one of some TEM modes for the lines shown in Fig. 2 (*A*, *B*, and *C*). The lighter areas in Figs. 3 correspond to more intensive fields, the areas where the field phase is 0 or π are highlighted by different colors.

IV. Calculation of Line-to-Line Junctions

The created algorithms of searching higher-order and TEM modes spectrum provide the possibility of calculation of the scattering matrix of a junction of two multi-conductor lines by the conventional mode-matching method. At calculating the needed coupling integrals, a preliminary analysis of reciprocal crossings of partial sub-domains of the enclosing line and the line being enclosed is carried out. In doing so, the coupling integrals values are defined as a sum of integrals over all the above-mentioned crossing areas.

V. Numerical Results

The cascaded assembling of the set of line-to-line junctions is done by the generalized *S*-matrix technique to obtain the full-wave model of the integrated circuit. The calculation of the frequency response does not require a lot of the CPU time because the coupling integrals for the needed line-to-line junctions are calculated once.

The response of the loop-element within the 120×14.5 mils dielectric-filled enclosure (ε=9.1) is shown in Fig. 4. The conductor thickness t=0.03 mils. The longitudinal size of the loop (between reference planes of input and output lines) is 120 mils.

More complex circuits have been also considered. One of them is a third-order low-pass filter (see Fig. 5). According to the proposed approach, it is segmented in twelve multi-conductor lines that generate eleven line-to-line junctions to be calculated. The filter is placed into the 128×36 mils enclosure. The values of t and ε are the same as for the above element. The frequency response of the filter is shown in Fig. 6. The computed response is close to that obtained by the EMSIGHT software based on the 2D moment method with roof-top basis functions. Such methods (EMSIGHT) cannot take into account a finite metal thickness and are extremely time-consuming compared to the proposed method. In particular, it takes about 6 hours of CPU time on a Pentium II/450 MHz PC for calculating 25 frequency points. The proposed approach allows to perform the low-pass filter analysis including the calculation of the lines' mode spectrums, coupling integrals for the line-to-line junctions (relative to 55-65 modes in the connected lines), and the response at 200 frequency points during 3.5 hours with the same PC.

VI. Conclusion

The proposed longitudinal decomposition approach gives a tool for an exact analysis of electromagnetic properties and potential of the microwave multi-layer circuits of various configurations. This approach has some advantages over the known approaches based on the method of moments and other direct numerical methods.

References

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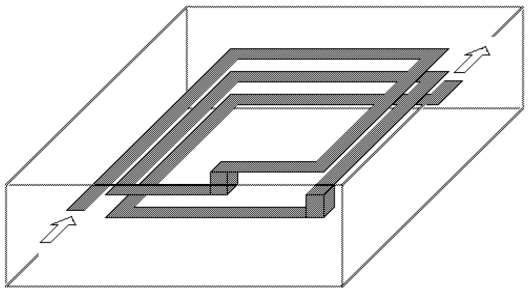

Fig. 1. Multi-layer loop-type element in a dielectric-filled rectangular enclosure.

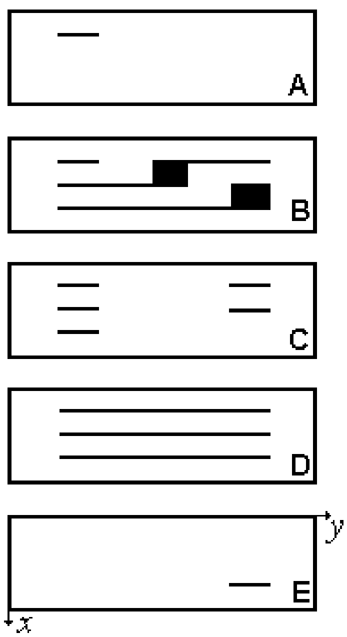

Fig. 2. Cross-sections of one- and multi-conductor lines forming fragments of the loop-type element.

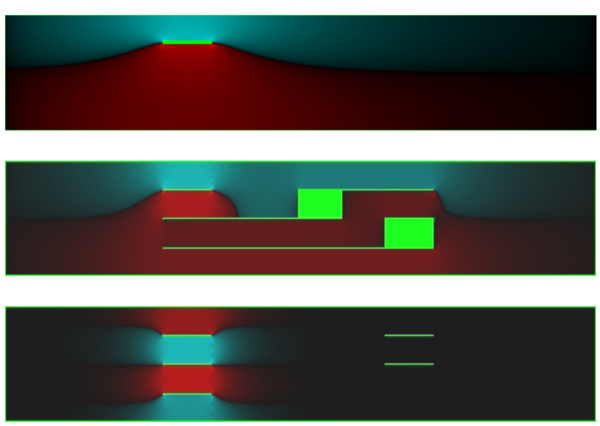

Fig. 3. Distributions of E_x-field components for some TEM modes in various lines.

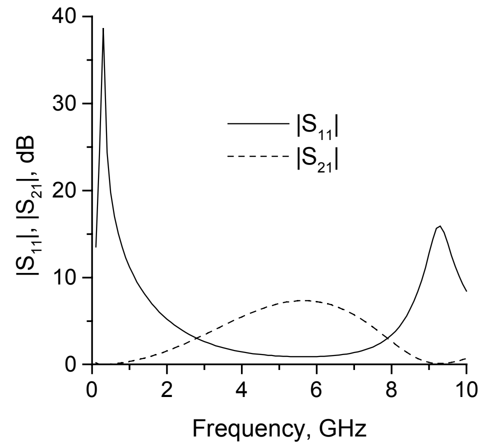

Fig. 4. Frequency response of the loop-type element.

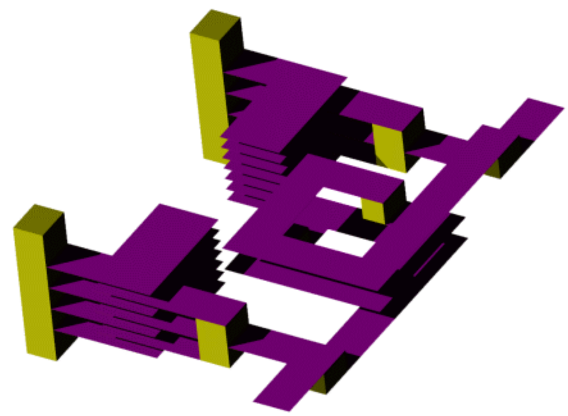

Fig. 5. The third-order multi-layer low-pass filter.

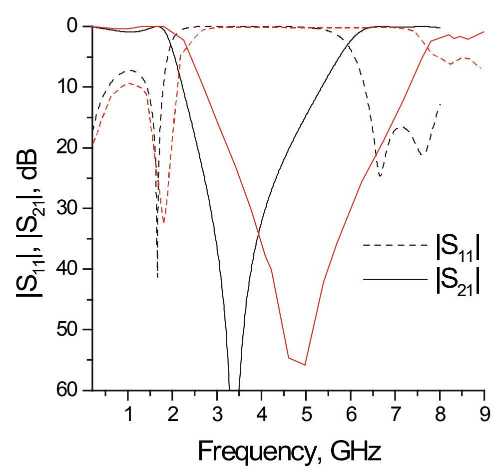

Fig. 6. Comparison of the results obtained by the present method (black curves) and by the method of moments (red curves).